WP 1: City typologies, climate impact functions and library for urban agglomerations

D1.4:
Library of adaptation cost and transition functions

Reference code: RAMSES – D1.4

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<th>Description</th>
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<tr>
<td>AEL</td>
<td>Annual expected loss</td>
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<tr>
<td>AP</td>
<td>Asset price</td>
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<td>CA</td>
<td>cellular Automata</td>
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<td>CB</td>
<td>Cusp-Bifurcation</td>
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<td>CBA</td>
<td>Cost benefit analysis</td>
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<td>CCS</td>
<td>Cusp-Control System</td>
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<td>DEM</td>
<td>Digital Elevation Model</td>
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<td>EEA</td>
<td>European Environment Agency</td>
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<td>EVT</td>
<td>Extreme Value Theory</td>
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<td>EWI</td>
<td>Early Warning Indicator(s)</td>
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<tr>
<td>HCS</td>
<td>Hopf-Control System</td>
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<td>LUCC</td>
<td>Land-Use and Cover Change</td>
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<td>MAS</td>
<td>Multi-Agent Systems</td>
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<td>NOI</td>
<td>Net operating income</td>
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<td>RCP</td>
<td>Representative Concentration Pathways</td>
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<td>SLR</td>
<td>Sea level rise</td>
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<td>UMZ</td>
<td>Urban Morphological Zones</td>
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Executive Summary

The term transition function describes an estimated function providing the likelihood of a transition depending on influencing factor(s). One can distinguish abrupt or gradual and favourable or unfavourable transitions. This deliverable tackles 3 dimensions of transitions in the context of cities in the climate change complex. The interdisciplinary library consists of an economics (Part I), geography (Part II), and physics (Part III) perspective. Parts I and II treat favourable transitions and Part III is about unfavourable ones.

In summary, Part I reveals how the generalised damage function is complemented by a mathematical representation of adaptation options and stakeholder behaviour. In the specific context of adaptation cost, transition functions are defined to express the probability of a transition from one state to a more adapted state. They are based on adaptation criteria that are derived from (expected) annual damage. The joint representation provides, for instance, an assessment of different adaptation options, the benefit of early investment, or the influence of cooperation among stakeholders.

In Part II, transition functions describe the probability that the urban system performs a transition from a non-urban state into an urban state. The part provides a library of standardised transition functions derived from spatially explicit regressions. The probability of such a transition depends on factors which describe the current condition of the system or the neighbourhood. Examples for such factors are distance to other urban systems, distance to coasts, or elevation. A convenient approach to this problem is the usage of logistic functions (sigmoid function) to describe the probability of a transition. The approach takes vulnerabilities of coastal regions into account (Part I) and opens the possibility to include probability decreasing factors, such as the hazard threshold (see RAMSES Deliverable 1.2).

In Part III an unfavourable transition is studied and how it can be avoided. As the considered system approaches the unwanted transition, the fluctuations increase which in turn is used to control the system so that it does not trespass the transition, but stays close. This closeness could be understood in economic terms, such as optimal efficiency. The results provide insights on the interplay between the time-scale for the system observation, the degree of sensitivity of the control feedback, and the intensity of the random perturbations in shaping the long-term control efficiency.

In summary, a library of standardised adaptation and transition functions has been assembled. Like a library of books, it is never complete but only represents a collection of important pieces. The database of generalised impact and transition functions links to RAMSES Work-Package 8, in which European urban strategies for transition are studied in terms of transition factors, modelling, and testing of alternatives. This deliverable also complements research done in Work-Package 8 in the sense that here theoretical foundations are explored.

The interdisciplinary approach of this deliverable is justified by the lack of a unique definition and the similarity to related concepts in the various fields. It can be concluded that the transition approach is a flexible concept to address the various areas where state changes are of interest. The probability of accepting (and undergoing) a transition can be represented as a function of net benefit, including uncertainty regions of risk averseness and negligence. An extension to transition matrices might be necessary in case the system can be in more than two states. Accordingly, a careful transition analysis can be recommended in the broader city context of adaptation, mitigation, and the win-win combination.
Introduction

In general, the term *transition* describes the shift of the considered system from one regime or state to another one (Scheffer et al., 2012, 2001; Walker et al., 2004; Carpenter and Brock, 2006; Livina et al., 2011). Such transitions can be abrupt or gradual and favourable or unfavourable. Favourable transitions in the urban context include for example a low carbon dioxide emission state or a state of adaptation to climate change intensified natural hazards (Reckien et al., 2015; Heidrich et al., 2013).

Usually, abrupt (i.e. large change in short time) and possibly unrecoverable transitions are meant. Nevertheless, transition can also be gradual. In case the state space is discrete (or comprises distinct stable states), the transitions must be abrupt. In case the state space is continuous (or approximately), the transition can be either abrupt or gradual. In particular, the term *transition function* describes an estimated function providing the likelihood of a transition depending on influencing factor(s).

It is important to view transition functions in the broader terminological context of tipping points, bifurcation, and critical thresholds. The expression *tipping point* originates in sociology and is commonly attributed to Malcolm Gladwell and Schelling (1971). Recently, the term has become a fashionable synonym in the natural sciences for critical points or turning points (and bifurcation), which usually have a negative connotation, i.e. unfavourable. *Bifurcation* describes a qualitative change of state as a function of (an) influencing parameter(s), i.e. a minor modification of the parameter(s) causes an abrupt change of the system. *Critical thresholds* occur in many different systems, such as in percolation theory, where the percolation threshold denotes the concentration at which the systems undergoes a transition from non-percolation to percolating (Bunde and Havlin, 1991; Stauffer and Aharony, 1994; Fluschnik et al., 2014). Thus, transition functions have various links to diverse disciplines around the natural and social sciences, and climate change impacts and adaptation take a special role treated in this deliverable.

This deliverable tackles 3 dimensions of transitions in the context of cities in the climate change complex. The interdisciplinary library consists of an economics (Part I), geography (Part II), and physics (Part III) perspective. Parts I and II treat favourable transitions and Part III is about unfavourable ones.

In Part I transition functions and generalised damage functions are employed theoretically to model adaptation. The economic value of urban landscape derives from the (potential) income that can be generated. Damage from coastal flooding diminishes income and may make one of several adaptation options attractive: protect, accommodate, or retreat (Klein et al., 2001). If urban structures cannot be maintained without the continual input of external funds, the abandonment of urban land use is a last resort.

For each adaptation option, transitions of state, such as the abandonment of urban land cover/use, are assessed via standardised probabilistic functions. Beyond the economic imperative, these adaptation functions allow for the inclusion of risk aversion as well as wanton negligence. The two counterparts represent the dominant source of uncertainty around the optimal adaptation decision. Extending on the damage assessment (RAMSES Del. 1.2), the resulting economic value after the consideration of sea level rise determines the probability for transition, e.g. the probability of abandonment. Adaptation costs of abandonment are given by the (potential) income that otherwise could be generated by the urban landscape.

Patterns of urban growth are being analysed in Part II in order to characterise future urbanisation. The investigation of growth patterns is done by means of land cover transition functions in the form of logistic regressions (sigmoid function) based on parameters such as existing urban areas, distance to shore, elevation, slope, etc. This library of standardised transition functions is being used to develop local urbanisation scenarios (Santé et al., 2010; Pontius Jr et al., 2008;...
Parker et al., 2003), down-scaled from large-scale projections. These scenarios allow estimating the additional urban area under risk of future inundation and the corresponding flood damage. Most important scenarios include “business-as-usual growth” or “avoiding low-lying areas” (complementing the theoretical part I). The provided parameters represent a database of transition functions for the considered case studies. The transition functions can describe reactive or proactive retreat-adaptation.

In Part III an unfavourable transition is studied and how it can be avoided. Exemplarily a system exhibiting a bifurcation point is considered. As the state approaches the unwanted bifurcation point, the variance increases which in turn is used to control the system so that it does not trespass the bifurcation point, but stays close. This closeness could be understood as a variant of self-organised criticality [see e.g. Turcotte et al. (2002) and references therein] and may be relevant in economic terms, such as optimal efficiency. The dynamics of a system subject to such a control process is analysed. The results provide insights on the interplay between the time-scale for the system observation, the degree of sensitivity of the control feedback, and the intensity of the random perturbations in shaping the long-term control efficiency.

The insights and results obtained within this deliverable link to RAMSES WP8, in which European urban strategies for transition are studied in terms of transition factors, modelling, and testing of alternatives. This deliverable also complements research done in WP8 in the sense that here theoretical foundations are explored from an interdisciplinary perspective.
Part I

Employing Transition Functions and Generalised Damage Functions to Model Adaptation

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1 Introduction

Damages from climate-related natural hazards are on the rise. While the increase is largely due to an increased exposure, global climate change is expected to have an intensifying effect on natural hazards. One example would be an increasing risk of coastal flooding due to the joint effect of seal label rice and altered cyclone activity (Boettle et al., 2013).

Conceptually, this issue may be addressed from two directions: i) via mitigation, where anthropogenic climate change is diminished by reduced carbon emissions, and ii) via adaptation, where assets at risk are propped to withstand the heightened risk from natural hazards. Whereas mitigation requires concerted action from all parts of society, adaptation has more complex mechanics, determined by local hazard risk and exposure, as well as individual stakeholder’s risk aware- and averseness.

In Deliverable 1.2 of the RAMSES project, we have demonstrated the top-down concept of a generalised damage function for climate-related hazards, applicable to direct damages emanating from, e.g., storms or coastal floods. By its very design, the concept incorporates the micro scale of the individual portfolio items and the macro scale related to an urban cluster or dedicated city. Bridging the scales, the concept is thus suited to incorporate various adaptation action.

In this chapter, we will reveal how the generalised damage function (Prahl et al., submitted) is complemented by a mathematical representation of adaptation options and stakeholder behaviour. In the specific context of adaptation, transition functions are defined to express the probability of a transition from one state to a more adapted state. They are based on adaptation criteria that are derived from (expected) annual damage.

The joint representation of damage and adaptation opens a wide range of opportunities for further analysis. It provides, for instance, a toolkit for the assessment of different adaptation options, the benefit of early investment, or the influence of cooperation among stakeholders.

We begin by giving a brief recap of the generalised damage function developed in the RAMSES Deliverable 1.2. In the subsequent section, we develop the mathematical representation of adaptation options, ranging from individual to communal action. We will conclude the chapter with an outlook into potential further research being enabled by the joint representation of damage and adaptation.
Figure 1: Schematic overview of the generalized damage function developed in RAMSES Deliverable 1.2. For a spatially delineated building portfolio as in (a), each building has a specific hazard threshold. (b) shows the frequency distribution of thresholds. For each of the coloured bars, damage occurs if the hazard magnitude exceeds this threshold. Based on the exceedance, the building damage is determined by the microscale damage function (c). Finally, the aggregate damage for the portfolio as a function of hazard magnitude is called macroscale damage function (d).

2 Generalized impact functions

This section provides a brief recap of the generalised impact function as presented in the RAMSES Deliverable 1.2. We begin by considering damages from coastal flooding of an idealised city. As illustrated in Fig. 1(a), the city is comprised by a spatially delineated ensemble of similar items, e.g. residential buildings. Considering direct damage only, the overall monetary damage in the city is equal to the sum over the damage costs for each individual building. Without loss of generality we assume equal monetary value for each item, which simply facilitates aggregation of relative (i.e. fractional) damages.

Neglecting ancillary damaging effects, such as floating debris, the damage to an individual building is dominated by the flood level at the site.

Considering topography, it becomes evident that individual buildings will be diversely affected according to their geographic location. Damage is caused only if the local flood level exceeds the surrounding ground level (orographic elevation) of the building. Taking into account natural and man-made barriers, the hazard threshold (i.e. the flood level at which a particular building is affected) will be correlated with the orographic elevation of the building ground floor. Thus, the city’s building portfolio can be described by a frequency distribution of hazard thresholds (Fig. 1(b)) that are derived from the elevation levels of each building and its surroundings.

A microscale damage function is used to estimate (relative) damage to those buildings, where
the local hazard magnitude exceeds the hazard threshold. This is shown schematically in Fig. 1(c). In the case of coastal flooding, we identify the exceedance as the inundation level of the building.

The average relative damage to all individual buildings determines the flood damage to the idealised city and, hence, constitutes the macroscale damage function (cf. Fig. 1(d)). Mathematically, Eq. (1) expresses the expected value of macroscale flood damage $d$ at flood height $x$ for the explicit and implicit portfolio description, respectively.

$$d_{\text{expl}}(x) = \sum_k f(\lambda_k)g(x - \lambda_k)$$

Here, $f(\lambda_k)$ denotes a frequency distribution of hazard threshold $\lambda_k$ and $g(x - \lambda_k)$ defines the microscale damage as a function of the exceedance.

Schematically, Fig. 1(b-d) shows the relationship between portfolio composition, microscale damage function, and macroscale damage function for a given hazard of magnitude $x_0$. For the colour-coded portfolio segments, the macroscale damage results as the frequency weighted sum over the microscale damages indicated by the respective coloured arrows.

3 Our Ansatz for Modelling Adaptation

The ansatz for modelling adaptation is complementary to that of the generalised damage function and as such draws upon the identical portfolio data. The modelling premise is that the individual stakeholders make decisions based on their individual risk prospect. Decision making hence presumes (statistical) knowledge of the peril and options for adaptation. The likelihood of making individual choices is governed by transition functions that are specifically defined for the context of adaptation. These describe the probability of making a transition (i.e. improving from one state to another via adaptation) subject to the perceived monetary constraints and the individual’s risk averseness. In the following, we describe each of these components and show how adaptation option can be modelled explicitly.

3.1 Portfolio Data

For modelling adaptation, we need to extend the basic portfolio data employed for the generalised damage function. So far, the data contain only topographic information about location and elevation of houses or blocks of houses. For adaptation, these data must be complemented by information about economic value, obtained either directly or by proxy. As explained in depth in the following Section 4, we employ the concept of net operating income (NOI) to relate property value to the capitalisation rate. The NOI is related to the asset price via the capitalisation rate. For modelling, we hence require the potentially achievable rent and either a direct estimate of the capitalisation rate or the asset price of a building or a representative building class. Expected annual damage cost that are implicitly included in the economic value can be inferred directly by using our damage function and hence require no additional portfolio information.

3.2 Risk Prospect

The adhoc risk prospect at a specific site is governed by the statistics for (extreme) water levels. Extreme Value Theory (EVT, e.g. Coles, 2001) can be employed to generate time series of extreme water levels. For example, a basic timeseries of extreme events could be generated from a simple Poisson point process in conjunction with a General Pareto Distribution. In combination with the
micro-scale damage function EVT can be used to obtain annual expected loss (AEL). Future risk prospects can be obtained either from adequate climate models or, rather simplified, by considering shifted sea levels according to sea level rise (SLR) scenarios. The latter method was employed by Boettle et al. (2013), who for instance elaborated the influence of SLR on AEL.

However, an individual stakeholder’s risk perception may be determined by alternative metrics, depending on his awareness of the situation. For example, possible metrics include the following:

Myopic: Risk perception is based on hindsight, considering only those water levels that have been experienced over a certain period of time.

Conscious: Risk perception is based on the statistical analysis of present flood levels. EVT is employed to anticipate extremes.

Omniscient: Risk perception is based on projected flood levels and considering EVT.

It is hence important to maintain the distinction between the actual AEL that is governed by EVT and the stakeholder’s risk perception, which may diverge accordingly.

3.3 Options for Adaptation

At each modelling step (e.g. annually), different management options should be evaluated for each individual building. The choice is of course dependent on the particular damage characteristics at the site and any precedent adaptation measures. From a general point of view, there are four fundamental management options:

Maintain: Business as usual. Damages are repaired to maintain the original state.

Accommodate: In-situ adaptation measures are put into place to lower overall damage costs.

Protect: Concerted action is taken to deploy large-scale protection measures (sea walls, dykes, etc.).

Retreat: Damage costs cannot be sustained. Site is abandoned and replaced by new construction in an area less exposed to flooding.

The first two are independent of the neighbour’s actions, while the latter may imply some concerted action. So for each option, a correlation parameter should govern the likelihood of collective choice. 0 correlation would imply autonomous adaptation, while a correlation of 1 would represent joint action.

If there is sufficient backing for a protective action, houses must re-evaluate their options based on their new risk profile as there may still be the need for further action. A protective action is undertaken if enough houses opt for this option.

3.4 Transition Functions in the Context of Specific Adaptation Options

A transition function describes the probability of passing from one (system) state to another. In the context of adaptation, a structure (i.e. the system) would evolve from its prior state to a state with a specific adaptation option in place. In its essence, the transition function that we employ in the context of adaptation is a probability distribution dependent on the net effect of avoided AEL and annualised adaptation costs. In the following, when we use the term transition function, we refer to a transition function in the context of a specific adaptation option.

In the simplest case, the transition function could be modelled by a Heaviside step function. It is however more realistic to assume individually differing transition points, leading to a smooth sigmoidal transition function. An illustrative example of a transition function is given in Fig. 2.
In general, the transition function should be centred around zero net benefit. The more the benefit increases, the more likely it should become to pick this adaptation option. A non-zero transition probability at negative benefit models risk averseness of the stakeholder. In this case, the stakeholder is willing to engage in an adaptation option even if his net benefit is negative. On the other side, there may also be negligence of an investment opportunity at positive net benefit. In other words, the stakeholder would be reluctant to engage in an adaptation option even if he would economically benefit from the measure. Negligence is reflected in the curve’s slow asymptotic approach towards certainty (i.e. probability 1) for increasing net benefit.

3.5 Modelling Cycle

The model set-up is that of an iterative process, where the economic value of each building and the stakeholder’s decisions are dependent on the previous modelling cycle. In damage modelling, annual time steps are a natural choice for the duration of one modelling cycle. The modelling cycle, depicted in Fig. 3, shows the modelling steps for an individual building.

The modelling cycle begins with the status quo of a new building, defined by its economic value and expected damage at the initial time step. In order to assess the need for adaptation, we forecast the building value based on the assumed SLR scenario and perception of the risk prospect (cf. Sec. 3.2).

The forecast building value allows for the assessment of the different adaptation options: protection, retreat, and accommodation (as defined in Sec. 3.3). Since the individual options of retreat and accommodation are conditional on communal protective action, the possibility of such protection must be assessed first. The check for the feasibility of large-scale protection is the only synchronous step for all individual buildings in the portfolio (see Sec. 4.3 for details). If protective action is undertaken, the situation for the individual building must be re-evaluated and the forecast updated before assessing the need of further individual adaptation.

After the assessment of protection, the second option for adaptation – accommodation – is assessed. The process to weigh different actions for accommodations and choose the most rewarding is described in Sec. 4.2.

Retreat is certainly the last resort and implies that a building can not be maintained without the continual input of external funds (as discussed in Sec. 4.4). If the conditions for retreat are met, a new building site is chosen and the modelling cycle is reinitialised.

When all adaptation options have been assessed, the building value is updated based on the
Figure 3: Conceptual illustration of the modelling process. For each iterative cycle (i.e. annual) an estimate of the future building value is calculated based on predicted damages. Subsequently, the criteria for the adaptation options protect, retreat, and accommodate are evaluated. Finally, the building value and cost/benefit accounts are updated for the next cycle.

chosen actions and the cycle is repeated. For the purpose of cost benefit assessment, the changes in AEL and the costs of adaptation are monitored for each time step. The details of such analysis are given in Sec. 5.2.

4 Modelling Cashflows and Transitions

4.1 Determination of Asset Value

The estimation of asset value and changes thereof is a crucial component of any adaptation assessment. Required is a simple means of relating property rent, annual damage cost, and cost of adaptation. This can be accomplished by use of the capitalisation rate, which relates the asset price (AP) or cost of capital with the net operating income (NOI),

\[ AP = \frac{NOI}{r} \]  \hspace{1cm} (2)

For a specific property, the NOI is comprised of the (potential) rent \( I \), expenses \( E \), and damage repair costs \( D \) (i.e. annual expected loss),

\[ NOI = I - E - D. \]  \hspace{1cm} (3)
Net operating income is affected by SLR and the employed adaptation. Let NOI\textsubscript{t} be the annual NOI at time \( t \). Then the cash-flow at time \( t + 1 \) is written as

\[
\text{NOI}_{t+1} = \text{NOI}_t - \Delta D_{\text{SLR}},
\]

with \( \Delta D_{\text{SLR}} \) and \( \Delta D_A \) being the changes in annual expected loss due to SLR.

Including the net benefit of accommodation, \( \Delta D_A - r C_A \), and protection, \( \Delta D_P - r \frac{C_P}{n} \), Eq. (4) becomes

\[
\text{NOI}_{t+1} = \text{NOI}_t - \Delta D_{\text{SLR}} + \Delta D_A + \Delta D_P - r C_A - r \frac{C_P}{n}.
\]

We assume that the adaptation costs \( C_A \) and \( C_P \) are subject to the same overall capitalisation rate as the original asset.

### 4.2 Adaptation Option: Accommodate

In-situ adaptation measures may be employed in order to accommodate to the adverse effect of SLR. However, an investment is only sensible, if the implied capitalisation rate on the construction cost \( C_A \) is equal to or exceeds the actual capitalisation rate. In other words, a net benefit of adaptation for the individual asset is required:

\[
\Delta D_A \geq r C_A.
\]

If several options for in-situ adaptation are available, the stakeholder will arguably choose the one that offers the greatest net benefit. Hence, while obeying the constraint given by Eq. (6), net benefit should be maximised such that

\[
\max [\Delta D_A - r C_A].
\]

Dependent on the risk-aversion of the stakeholder, an investment may be favourable even if the net benefit is slightly negative. Here we employ a sigmoidal transition function that allows to incorporate risk aversion and potential negligence of an adverse monetary impetus (cf. Fig. 2). Defining a sigmoid transition function \( \Phi \), the probability of applying the accommodate option is given as

\[
P_A = \Phi(\Delta D_A - r C_A).
\]

In general, the decision for accommodation is conditional on the prior decision on protection.

### 4.3 Adaptation Option: Protect

When considering the need for protection, we must consider not only the damage reduction for the individual structure but also the commons provided by the city. In order to consider the value of the commons, we include an overall value \( V \) into our set of equations. So, effectively, every household has a stake in the decision for large-scale protection.

Large-scale protection measures, such as sea-walls, offer protection not only from direct damages but also mitigate indirect damage costs that have not been included in the employed damage function. Post-disaster research indicates that indirect socio-economic losses potentially surmount direct losses from property damage (Hallegatte et al., 2007). Furthermore, Koks et al. (2015) show that direct loss typically dominates EAL, whereas indirect losses strongly increase for extreme events. While these effects are not included in the present approach, it is worthwhile to note that its inclusion would expedite the implementation of protective measures.
Regarding the economic impact of protection from a city perspective, implementation of a protection measure requires a positive net benefit such that

$$\sum \Delta D_P + V \geq r C_P,$$

where $\Delta D_P$ represents the avoided damage per building.

Each protection measure or rather any desired protection height comes at a specific cost of protection. Arguably, it may be assumed that the community will generally choose the option that offers the greatest net benefit. This leads to a simple maximisation

$$\max \left[ \sum \Delta D_P + V - r C_P \right]$$

while obeying the constraint of Eq. (9).

Similarly to the case of accommodation, an investment in protection may be favourable even if the net benefit is slightly negative (risk aversion). Without risk aversion, we would expect the transition function to be a Heaviside step function. However, taking into account risk aversion and potential negligence of an adverse monetary impetus a sigmoidal transition function as in Fig. 2 is more realistic. Defining a sigmoid transition function $\Phi$, the probability of applying the protect option is given as

$$P_P = \Phi(\sum \Delta D_P + V - r C_P).$$

This approach of employing a global transition function for the city implies a city decision-maker – a benevolent planner in economic terms. Arguably, this assumption can be justified on the city level, where detailed hazard information is typically available (e.g. via flood risk maps) and city planners are obliged to make their decisions for the better of the citizens.

The situation of having the decision on adaptation on single household as well as on city level gives rise to a free-rider problem. In speculation on the construction of flood protection, individual stakeholders could postpone or even avoid investing in local adaptation options. The incentive for such behaviour would be to externalise potential adaptation cost to the community, thereby reducing the free-rider’s own burden. It is, however, beyond the scope of our simple framework to assess the free-rider problem. In our approach, stakeholder decisions are based solely on the state of their own property and their expectation of the particular flood risk.

Avoiding the concept of a benevolent planner (but not the free-rider problem), an alternative stochastic modelling approach could be employed to model a complex decision process. In this case, the individual transition probabilities $P_i$ for all affected houses $i$ are calculated. Based on an imposed interaction function $\omega_{i,j}$ (i.e. correlation between the decisions of stakeholders $i$ and $j$) we can then determine a covariance matrix and generate correlated binomial random numbers. For these numbers 0 represents no change and 1 represents implementing an adaptation option. For the evaluation of joint action the overall number of houses buying into the scheme is determined. Now, either a majority vote is taken, or the sum of averted damages for all positive voters is weighed against adaptation cost less the value of the commons. In this scheme, the interaction function allows to model cooperation, e.g. with neighbouring stakeholders.

In contrast to the remaining adaptation options, accommodate and retreat, protection may not take immediate effect. For simplicity, however, we assume direct implementation of the protective measure. Otherwise, one could accumulate EAL until full implementation and add to the overall cost of protection in Eqs. (9-11). In any case, further adaptation options evaluated at the same time step should consider losses above the protected level only, in order to avoid artificial redundancy.
4.4 Adaptation Option: Retreat

A retreat would take place at latest if the additional loss due to SLR exceeds the buildings NOI including potential adaptation benefits. In mathematical terms

\[ \Delta D_{\text{SLR}} \geq \text{NOI} + \Delta D_A + \Delta D_P - rC_A - r \frac{C_P}{n}. \]  

(12)

The probability of abandonment \( P_{\text{abandon}} \) is given by a transition function \( \Phi \) subject to the above condition,

\[ P_{\text{abandon}} = \Phi(D_{\text{SLR}} - \text{NOI} - \Delta D_A - \Delta D_P + rC_A + rn^{-1}C_P). \]  

(13)

If a site has been abandoned, the destination for retreat should obey two conditions: First, it should not be subject to significant SLR damages. And second, it must be a likely location (high probability of urban sprawl). To meet the first condition, we assume that a new house is built only if its likely net operating income \( \text{NOI}^* \) (including the anticipated damage costs \( D^* \) at the new site) exceeds the annualised building cost \( rC_B \),

\[ \frac{I - E - D^*}{\text{NOI}^*} - rC_B \geq 0. \]  

(14)

Again, we employ a custom transition function \( \Phi \) to describe the probability of transition \( P_{\text{trans}} \) from an empty plot of land to built up area,

\[ P_{\text{trans}} = \Phi(\text{NOI}^* - rC_B). \]  

(15)

The larger the house’s value compared to the building cost, the more likely is the transition.

In order to meet the second condition, one must consider the spatial probability of urbanisation within the perimeter of the city. To this end, patterns of urban change must be identified by remote sensing, urban planning, or otherwise. In Part II we investigate these patterns in detail, employing a logistic regression based on parameters such as existing infrastructure, distance to shore, density, elevation or slope. This procedure allows for the estimation of a site-specific probability of urbanisation \( P_{\text{urb}} \).

Finally, we can write the probability of constructing a new building as the product of the probabilities of urbanisation and transition,

\[ P_{\text{new}} = P_{\text{trans}} P_{\text{urb}}. \]  

(16)

The probability of constructing a new building is of course relevant not only for retreat, but for the allocation of city growth in general. By this means, it would be feasible to incorporate different scenarios for city growth into the assessment.

5 Application and Outlook

5.1 Parametrisation

Model parametrisation is a crucial step in the estimation of adaptation benefit and the evaluation of different adaptation options. Along the modelling chain, the following parameters would be required:

**Damage function**: Relative depth-damage function for single structures.
**Portfolio Valuation:** Property values; capitalisation rates; discount rates (required for cost-benefit assessment, see Sec. 5.2).

**Transition functions:** Functional shape; estimates of the degrees of risk aversion and risk negligence.

**Adaptation options:** Cost of adaptation options; effect on the depth-damage function for single structures (accommodate); design flood level (protect); probability of urbanisation (retreat; see Part II).

Depending on the focus of the analysis, parametrisation could the simplified by reducing the complexity of the general model. For instance, neglecting risk aversion and negligence, complicated transition functions could replaced by simple Heaviside step functions. Another potential simplification could be the reduction of adaptation options.

### 5.2 Cost-Benefit Assessment

Our ansatz for modelling adaptation allows for a straight-forward implementation of cost benefit assessment (CBA). It provides a means to compare different adaptation strategies, as well as different modes of risk perception and risk taking behaviour (Boardman et al., 2010).

For instance, the way in which flood risks are perceived within a community may have significant impact on the timing of adaptive actions. Thus a myopic risk perception could imply delayed action compared to a omniscient view. However, only CBA may answer how much net benefit an altered risk perception, i.e. via information campaigns or otherwise, has to offer.

In order to enable CBA, it is necessary to monitor changes in EAL as well as additional costs for adaptation. Changes in EAL are driven either by SLR or by averted losses due to the various adaptation measures. Having established cost accounts for the aforementioned costs, the model allows a CBA both on the microscale for specific buildings and on macroscale for a designated case study city.

CBA should in general be based on the present value of cash flows and as such requires discounting of cash flows. This aspect necessitates the assumption of meaningful discount rates, as these can have a marked effect on the evaluation of competing scenarios.

### 5.3 Outlook

On a city level, adaptation is the result of a complex interplay of many actors with different risk exposure, risk perception, risk taking and adaptation strategies. On the foundation of a generalised damage function, we have established a means to evaluate the net benefit of adaptation, which allows for the identification of an optimal adaptation strategy.

However, this strategy is dependent on the model forcing, i.e. the assumed scenario for SLR. Undertaking a leap of faith, it may be possible to break down the cost of mitigation, i.e. for containing anthropogenic climate change and the consequential SLR, onto the level of urban agglomerations/cities. Under this supposition it may be feasible to undertake a comparison of the cost/benefit of mitigation and adaptation.

While an overview of win-win situations between mitigation and adaptation for a specific natural hazard such as coastal flooding may be at grasp, we must however acknowledge the different political decision processes behind mitigation and adaptation, which determine the leeway for action.
Part II
Urban Transition Functions

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6 Introduction

Urban growth is a key challenge for the 21st century. Not only regarding building new infrastructure, but also including climate change. The growth of an urban system has always to consider adaptation and mitigation aspects for a sustainable growth. Therefore it has to be known where are possible spots for future urban systems and where are places threatened by future impacts, such as sea level rise.

Land-Use and Cover Change (LUCC) modelling tries to answer the first aspect and there are multiple approaches to do so. One common approach are cellular automata (CA), which are based on a two-dimensional grid and have fixed rules how a state of a cell within this grid will change for the next time step depending on the state of neighboring cells. CA are simple, flexible and can easily deal with the spatial and temporal dimension of a change process (Santé et al., 2010). However, finding the right set of rules for a cellular automata is a highly complex and critical task. Another common approach are Multi-Agent Systems (MAS). Agent-based modelling can combine the two-dimensional grid of a CA with agent based decision making and interaction between different agents at the same or a different location (Parker et al., 2003).

A more simple and ground proofed approach is spatial explicit statistical regression (Lin et al., 2011), which where applied to multiple regions (López and Sierra, 2010; Serneels and Lambin, 2001) and is also used in multiple LUCC-models to model urban growth. For example in SLEUTH model used by Silva and Clarke (2002) for the region of Lisbon, Portugal, or Veldkamp and Fresco (1996) using the CLUE model to simulate land-use change for the complete region of Costa Rica. Statistical regression uses the observed relationship between variables of a system, or in other words the observed change depending on different variables, to predict the future state of the system. Therefore historical data of urban growth is needed.

For future projections not spatial explicit studies of urban growth, such as the estimations of global urban expansions per country by Angel et al. (2011), or the projections of urbanisation for the Shared Socioeconomic Pathways by Jiang and O’Neill (2015) or also spatial explicit projection with a coarse resolution such as Hurtt et al. (2011) can be used as exogenous driver for a spatial explicit high resolution projection.

This section provides a library of standardised transition functions derived from spatially explicit regression. In this context a transition function describes the probability that the urban system performs a transition from a non-urban state into an urban state. The probability of such a transition depends on factors which describes the current condition of the system or the neighbourhood. Examples for such factors are distance to other urban systems, distance to coasts or elevation. A convenient approach to this problem is the usage of logistic functions (see Fig. 4, to describe the probability of a transition.
Figure 4: A logistic curve as described by Eq. (17). The logistic curve can be used to describe the probability for a transition from a non-urban systems to an urban systems. The probability for a transition ranges from 0, exceptionally unlikely, to 1, virtually certain.

7 Method

The following method describes a top-down approach to obtain probabilities of future urbanisation, with the help of logistic regression. Therefore, we use observations of previous changes in land-use from non-urban to urban to fit a regression curve. Assuming that the reasons for a transition to an urban state do not modify with time, we can use the identified regression curves to compute the probability of future transition.

The presented method can easily deal with large scale high resolution, 25-100m, remote sensing data sets and can be applied globally and to various scales.

7.1 Concept of Transition Functions

Each single cell in the data set is considered as a system described by different variables such as, distance to other urban areas or elevation and can have two states urban or non-urban. The transition curve between these two states will be given by a logistic curve, which can be derived from a binomial logistic regression using the describing variables as predictor variables. Statistical Regression is a proofed method to model land-use change (Pontius Jr et al., 2008) and can be obtained across all scales.

Using the land cover change from non-urban to urban for the years 1990 to 2006 allows to fit logistic curves for the past urban growth. In this context the identified transition function then can be used to compute the probability of a future transition from a non-urban to an urban state for current conditions, meaning all prediction variables are based on the year 2006.

The long observation period of 16 years, from 1990 - 2006, shows the highest amount of urban change, see section 8.1. This is important, as the fit is more robust for a higher signal (Pontius Jr et al., 2008). We consider only change from non-urban to urban. Only non-urban cells in 2006 are considered for the future projections. But a generalization is possible.

7.2 Binomial logistic regression for identifying Transition Functions

Binomial logistic regression is a method for fitting a regression curve, \( P(z) \), when \( P \) is a categorical variable. In our case the state of the system or to be more precise the state of a land-use cell, urban or non-urban. Where 1 denotes an urban and 0 a non-urban system state. The probability
of a transition from one state to another can be described by a logistic curve. Figure 4 illustrates a logistic curve, which is given by

\[ P(z) = \frac{1}{1 + e^{-z}}. \]  

(17)

The variable \( z \) is a polynomial representing a set of predictor variables \( x \).

\[ z = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots \beta_k x_k \]  

(18)

Predictor variables are those variables which can be used to explain the state of a system. Based on a review of different LUCC models (Pontius Jr et al., 2008) we choose the smallest distance to other urban cells, the distance to nearest major roads, the smallest distance to coast, the elevation and the slope as possible predictor variables. Each predictor variable is described by a coefficient \( \beta \). The sign of the coefficient highlights the relationship between \( P \) and the prediction variable \( x \). For a positive \( \beta \) \( P \) increases with increasing \( x \) and for a negative coefficient a increasing predictor variable results into a decreasing probability of a change from non-urban to urban.

Not all prediction variables may be needed for the best prediction. To identify the best model a stepwise bidirectional selection, which is a forward selection with the possibility to deleting variables, were performed. In detail the selection starts with no variables included in the model. Then it adds in variables according to their importance until no other important variables are found. Additionally it checks every step for obsolete variables and erases them. Here we measure importance by the Akaike’s Information Criterion (Akaike, 1998).

For fitting the regressions the observed change of land-use from the year 1990 to the year 2006 will be used, considering all prediction variables for conditions in the year 1990. With the identified transition functions it is possible to compute the probability of a future transition of a land cover cell based on the current conditions with Eqs. (17) and (18). Of course the data set for the smallest distance to the next urban cell differs for the years 1990 and 2006 due to the urban growth and consequently we use one data set based on the year 1990 and one based on the year 2006 specifying the smallest distance to other urban cells for all grid cells. The final results are probability maps for future urbanisation, see Fig. 7.

To consider regional differences this approach will be applied to small tiles defined by a masking grid equivalent to the one used for RCP scenarios. The Representative Concentration Pathways (RCPs) represent emissions, concentrations, and land-cover change projections for the IPCC AR5. The harmonized future land-use projections from each RCP (Hurtt et al., 2011) allowing also future projections. This dataset represents land-use transitions annually for the RCPs from 2005 to 2100 at 0.5° times 0.5° resolution.

8 Data

CORINE Land Cover data (European Environment Agency (EEA), 2014a) was used as basis to observe the urban growth. Overall there are three different distance datasets used for the analyses. First the distance to other urban cells, where urban cells are defined based on the Urban morphological zones 1990 (European Environment Agency (EEA), 2014b), second the distance to roads based on the GRoads-Dataset (Center for International Earth Science Information Network (CIESIN), 2013), and third the distance to coast based on the EU-coast data (European Environment Agency (EEA), 2013). In each distance data set the value of a cell represents the nearest distance to the measured variable. The EU-Digital Elevation Model over Europe (EU-DEM) (European Environment Agency (EEA), European Commission, 2013) was used for elevation and slope data and completes the five input layers. These five input layers a derived by three different data-sets which are described in more detail in the following sections.
8.1 Land-use data
CORINE Land Cover data (European Environment Agency (EEA), 2014a) is available for the years 1990, 2000, 2006 with a resolution of 100m. However, the spatial extent is different and not all EU and associated countries are represented in each time step. For Example the United Kingdom is missing in the 1990 data set. Overall for the year 1990 are 26 (27 with late implementation), for 2000 30 (35 with late implementation) and for 2006 38 countries part of the data set.

The data set represents 44-different land cover classes, which are categorized into five groups, artificial surfaces, agricultural areas, forest and semi natural areas, wetlands and water bodies. The artificial surfaces group contains eleven classes, including urban fabric, urban green areas, industrial areas but also mine, dump and construction sites. For a clear distinction between urban and non-urban classes (not all industrial areas may be urban, on the other hand some water bodies belong to the urban system) we are using the concept of the Urban Morphological Zones (UMZ)(European Environment Agency (EEA), 2014a). UMZ are defined based on the Corine land cover classes as a set of urban areas laying less than 200m apart and taking into consideration the urban tissue and function. The result is a data set with a resolution of 100m containing only zeros for non-urban cells and ones for urban cells. The UMZ data were also used to generate the distance data set to the next urban cell for the year 1990 and 2006.

8.2 Major roads
The Global Roads Open Access Data Set, Version 1 (gROADSv1) (Center for International Earth Science Information Network (CIESIN), 2013), combines available public domain roads data by country into a global roads coverage. The data was compiled from multiple sources and with most uptodate data from 2010. The data set were compared to roads found in Google Earth imagery, showing accuracy of 30m to 500m. Used sources are for example: Data generated from ASTER imagery for NASA SERVIR project, Ordinance Survey (UK), Open Street Map or VMAP-1. Based on the gROADS data set we created buffers around all roads to compute a input layer containing the smallest distance to a major roads for every cell in the CORINE Land Cover data.

8.3 Digital elevation model
The EU-DEM (European Environment Agency (EEA), European Commission, 2013) a European digital elevation model with a resolution of approximately 25m. It is is a hybrid product based on SRTM and ASTER GDEM data, fused by a weighted averaging approach. This data set was used to obtain the elevation of each cell in the CORINE Land Cover data. Comparing the elevation of the four neighbouring cells (rook case) according to Ritter (1987), additional the slope of each land cover cell were derived from the EU-DEM.

9 Results
The top-down approach was applied to two RAMSEs case-study cities, Antwerp and Bilbao. Based on the previous introduced RCP-grid not only the grid-cell which contains Antwerp, respectively Bilbao, but also the eight neighbouring grid-cells, further called tiles, were considered, see Fig. 5. Both cities are located in the center grid-cell, representing by tile 5. Each tile is approximately 55 times 55 km in size.

The region of Antwerp is characterized by a highly dens urbanised area. With tile 2 containing Rotterdam and The Hague in the north and tile 8 in the south containing Brussels there are some big cities in direct neighbourhood. Tiles 1, 2 and 4 have direct access to the sea, where tile 1 is mostly ocean.
Bilbao and its neighbourhood is not so highly urbanised. Other big cities are Santander in tile 4 and Vitoria in tile 8. Tiles 1, 5 and 6 are part of the Spanish coastline. Also here tile 1 consist mostly of ocean. Tiles 2 and 3 are not containing land at all and are not taken into consideration for further analysis.

For each of the 18 tiles the logistic regression provides transition functions (see Fig. 6) and coefficients (see Tab. 1 and 2). A positive coefficient means that with higher values of the prediction variable the probability increases, and the opposite a negative coefficient indicates a shrinking probability for a future transition with increasing prediction variable.

9.1 Detailed overview of identified Transition Fractions

Overall, urban distance are identified in 16, road distance in 15, slope in 14, elevation in 13, coast distance in 9 of 16 tiles as important factors. In all tiles of Antwerp and Bilbao the coefficients for the prediction variables urban distance and road distance are negative, indicating a increasing probability of transition with smaller distances. Road distance is only in tile 7 in Bilbao not considered. All coefficients for the distance to coast in Bilbao are positive and only in tile 7 and 8 not relevant, which is plausible due to the fact that they have the largest distance to the coastline. In Belgium the distance to coast is only considered in 4 areas (1, 3, 4, 5) and only in tile 4 it is negative. Only the elevation coefficient in Bilbao’s tile 7 is positive. All slope coefficients for the region of Bilbao are negative, whereas in the region of Antwerp three negative coefficients (in tiles 6, 7 and 9) and five positive coefficients (in tiles 1, 2, 4, 5 and 8) are obtained.

Antwerp Full set of prediction variables are used in Antwerp’s tiles 4 and 5, see Tab. 1. Tiles 6, 7 and 8 far from the coast are missing only the distance to coast, but also the coastal tile 2 with The Hague and Rotterdam is missing only the distance to coast. For the mainly by ocean dominated region 1 the only not-considered variable is the elevation. Tile 3, with Utrecht, missing only the slope variable. The only region within Belgium case area with less than 4 prediction variables, considering only urban distance, coast distance and slope, all three with negative coefficients, is tile 9. The four tiles 1, 2, 4 and 5, which have the smallest distance to the coast, have all positive coefficients for the prediction variable of slope. However, these regions are all very flat land with
<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>Urban distance</th>
<th>Road distance</th>
<th>Coast distance</th>
<th>Elevation</th>
<th>Slope</th>
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<td>-0.051</td>
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<td>NR</td>
<td>-0.201</td>
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Table 1: From the logistic regression derived coefficients $\beta$ for the city of Antwerp for all nine tiles, see Fig. 5. Coefficients for urban and road distance and $\beta_0$ are in all tiles negative, whereas the other prediction variables have mixed signs. Not relevant prediction variables are indicated by NR. Those variables were not able to improve the model, measured by the Akaike’s Information Criterion. The coast distance were considered as relevant in just four tiles and represent the least relevant prediction variable.

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>Urban distance</th>
<th>Road distance</th>
<th>Coast distance</th>
<th>Elevation</th>
<th>Slope</th>
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<td>-0.253</td>
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</table>

Table 2: From the logistic regression derived coefficients $\beta$ for the city of Bilbao for all nine tiles, see Fig. 5. For tiles 2 and 3 no coefficients exists, because these tiles consists only of ocean. The incremental $\beta_0$ and the coefficients for the smallest distance to other urban systems, the smallest distance to roads and the slope are all negative. Whereas the prediction variable distance to coast is described only by positive coefficients. Not relevant prediction variables are indicated by NR. Those variables were not able to improve the model, measured by the Akaike’s Information Criterion.
a maximum slope of $16^\circ$.

**Bilbao** Full set of variables are considered for tiles 4, 5, 6 and 9 of Bilbao, see Tab. 2. In the flat coastal and mainly by ocean dominated tile 1 only the prediction variable slope is missing. tile 7, which is the tile with the largest distance to the coast, missing the distance to roads and coasts and is the only tile with a positive elevation coefficient, indicating that higher areas are more likely to become urban. But it is also the tile with the smallest coefficient, $\beta_{\text{slope}} = -0.55$, for the prediction variable of slope. So it is more sensitive to the orography or the roughness of the region. In tile 8 prediction variables coast distance and the elevation are not relevant and consequently not considered.

### 9.2 Probability maps of future urbanisation

Figure 7 shows the probability of a future transition from a non-urban to a urban state of a cell. The probability ranging from 0 to 0.3, where the theoretical maximum probability could be 1.

In the region of Antwerp the areas with highest probabilities, with around 0.3, are located in tiles 2, 3 and 6 including the peri-urban areas of The Hague, Rotterdam and Utrecht. The patches of probability are spread widely, getting less dens from north to south. The most unlikely urbanisation can be expected in tile 7, the region of Gent.

At the Spanish coast around Bilbao the probability patches of a future urbanisation are more compact than in Antwerp. Regions with the highest urbanisation probability of around 0.2 can be observed at the coast around Santander, around Bilbao and in the peri urban areas of Vitoria. There are some small spots with low probabilities around 0.05, but the most areas have around zero probability for future urbanisation.

### 9.3 Adaptation Options: Controlled growth

So far we only considered transition probabilities from non-urban to urban based on observed changes, assuming constant rules for urban growth behaviour. However, as mentioned before a sustainable city growth should consider climate change and we will now adapt our method to the future risk of coastal flooding, taking the vulnerability of coastal regions into account, see Part 1. This can also be understood as a kind of controlled growth policy or land-use planning. In the context of fluvial and coastal floods land-use planning is a proven method to manage or minimize damages of the urban infrastructure (Kreibich et al., 2015).

For illustrative purposes we assume that the probability of transition, Eq. (16), uniquely depends on the hazard threshold $\lambda$ (see Part I of Deliverable 1.2). In the case of coastal floods, the hazard threshold is given by the minimum flood level for which a site is inundated. The exact functional form of $P_{\text{trans}}$ is unknown and we assume the relation as depicted in Fig. 8, a linear increase until a saturation level. It captures that sites with low hazard threshold, i.e. those which are more likely to be flooded, have reduced probability to develop to urban land-use. Figure 9 shows the probability of a transition to urban state, including $P_{\text{trans}}$ for a 10m flood scenario. Compared to the free growth shown in Fig. 7 a noticeable decrease of the transition probability for the coastal regions of Antwerp and surrounding occurs. The flat coastal areas have now, probabilities of around 0 to 0.05 instead of values up to 0.3 without the consideration of the hazard threshold $\lambda$. The more inland areas stays untouched. In Bilbao the changes are not so drastically. Only small sites on the coastline are affected, mainly focused around the bay of Santander.
Figure 6: Transition functions for each predictor variable, keeping all other variables constant to 0. The different colors representing the single tiles, introduced in Fig. 5. Blue-green colors refer to Antwerp and yellow-red colors to Bilbao. Bilbao 2 and 3 are missing, because they consist only of ocean and are not considered in the analysis.

All curves for the urban distance are falling with greater distances. In Antwerp the probability increases rapidly for distances less than 10km. Transition functions for the distance to roads are not so curved, but still with negative coefficients, indicated by falling probabilities. Coefficients for the distance to coast varies in sign but have almost a constant trend, exclude tile 1 in Antwerp. However, the s-curved function is a bit misleading due to the fact that in this tile the largest distance to the coast is just around 6km and so just the first steep part of the curve up to a probability of around 0.3 is relevant here. Elevation curves have more constant trend, whereas the shape of the transition functions for the variable slope are highly diverse.

Due to the high $\beta_0$ of around 0.61, see Tab. 2, the curves of Bilbao tile 9 are for all five variables a bit separated from the others.
Figure 7: Probability maps for future transition from urban to non-urban. Abrupt changes of probability on the grey dashed lines are due to different regression models used for the different tiles. See Fig. 5 for the introduced tiles respectively Fig. 6 for the used transition functions.

Figure 8: $P_{\text{trans}}$ as a function of hazard threshold $\lambda$, see Part I of Deliverable 1.2. In our case $\lambda$ describes the flood height and is measured in meters.
Figure 9: Probability maps for future transition from urban to non-urban state, including $P_{\text{trans}}$ as a function of the hazard value $\lambda$. Compared to Fig. 7 the probability of urbanisation in the coastal regions of Antwerp, especially the surroundings of The Hague, Rotterdam and Utrecht decreases drastically. The urbanisation probabilities for Bilbao show just minor decreases.

10 Discussion & Outlook

The presented method is able to identify transition functions and use them for computation of future probabilities for a state change from a non-urban to an urban system. It can handle huge data sets and can be used to describe urban growth in different regions.

The introduction of tiles, to include regional characteristic, proofed as valuable. The results show complete different regression models for neighbouring tiles. Without these tiles the regional characteristic would be lost. However, the influence of the tile size is not probed. Another valuable point is the possibility to include probability decreasing factors, such as the hazard threshold, see Part I of Deliverable 1.2.

The method of transition functions, as a describing tool for urban growth, can be extended to include socio-economic factors as prediction variables and can lead to a more complex understanding of the urban growth process. Probability maps of urbanisation, based on the identified transition functions, can be turned into urban growth projections using the harmonized future land-use projections from multiple RCP-scenarios (Hurtt et al., 2011) as exogenous drivers. This will lead to a similar downscaling approach as presented by Verburg et al. (2008). However, instead of using a complex LUCC model we obtain the rules of change direct from observed ground proofed data with a hundred times higher resolution.

Additionally the introduction of other hazard thresholds could be used to evaluate future growth scenarios and adapt to future conditions. Following a more controlled growth policy can reduce risks before they raise.
Part III

Variance-based control of regime shifts: bistability and oscillations

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11 Introduction

A variety of real world and experimental systems can display a drastic regime shift, as the evolution in one its parameters crosses a threshold value. Assimilation of such a transition with a bifurcation has allowed to identify so called “early warning signals”, at the level of the time series generated by the system underscopic. The literature in early warning detection methods is currently expanding and their potential for practical applicability is being discussed in different contexts. In this work, we elaborate on the use of the variance of a system variable, which constitutes the simplest early warning indicator, to gain control on the long-term dynamics of the system, while extending an exploitation phase. In particular, we address the cases of the cusp and Hopf normal forms, as prototypical examples of bistability and oscillations. Our results provide insights on the interplay between the time-scale for the system observation, the degree of sensitivity of the control feedback and the intensity of the random perturbations, in shaping the long-term control efficiency.

Along the last few decades an increasing number of scholars has been contributing to develop a coherent and further integral body of knowledge on complex real world systems. Cross-fertilization of concepts and tools stemming from dynamical systems, statistical physics and information and computation theories has allowed to identify and analyse systems whose observable behaviors result from similar underlying mechanisms (Nicolis and Prigogine, 1989; Badii and Politi, 1999). As a result of these attempts, a key finding has been realization of the ubiquity of nonlinear feedbacks in the dynamics of different classes of systems.

In particular, the occurrence of drastic, qualitative shifts of the functioning regime of systems –as resulting from strong positive and negative feedbacks– has attracted interest in diverse research fields, as diverse as microscopic physical systems at equilibrium (Stanley, 1971) and their macroscopic non-equilibrium counterpart (Nicolis and Prigogine, 1977; Cross and Greenside, 2009), ecology (Scheffer et al., 2001; Scheffer, 2009), socio-physics (Castellano et al., 2009) and Earth system science (Lenton et al., 2008; Scheffer, 2009).

Insights into the phenomenology, related to regime shifts and the development of new methods for their early detection, have relied on minimal models. A frequent idea in this approach is to cast the description of the system evolution as a stochastic differential equation (Gardiner, 2010), of the form:

\[
dx = \mathbf{F}(\mathbf{x}, \lambda)dt + d\mathbf{W}_x
\]

(19)

The key variables of the system are represented as a vector \( \mathbf{x} = (x_1, x_2, \ldots, x_N) \) and their dynamic interrelationships are considered as deterministic laws, described as a vector field \( \mathbf{F} \). Vector \( \mathbf{W}_x \) aims at introducing the effect of incessant perturbations on the \( \mathbf{x} \) components, as typically resulting from endogenous meso-scale dynamic complexity and from an infinite number of uncorrelated environmental factors. Consequently, \( \mathbf{W}_x \) is assimilated to a Gaussian Wiener process (Gardiner, 2010) with standard deviation \( \omega_x \). The parameter \( \lambda \) controls the qualitative behavior of solutions
to the deterministic part of system (19) around a stationary state $x_s$. In this framework, a regime shift is represented as a local bifurcation occurring in the deterministic part of the system at a critical point $(x_s, \lambda_c)$, i.e., that the Jacobian matrix of $F$, evaluated at $x_c$, possesses one eigenvalue whose real part tends to zero as $\lambda$ approaches the value $\lambda_c$ (Guckenheimer and Holmes, 2013). Thereby, the response of the system to random fluctuations can be studied along the evolution towards the bifurcation point.

Given the relevance of applications and implications of regime shifts in real world systems, it is of increasing interest to construct early warning indicators (EWI) (Scheffer et al., 2009, 2012) – with focus on identifying statistical signatures of the loss of stability in the time series generated by the evolution of the system towards criticality. Conjointly, a relevant phenomenon is the slowing down of the fast stabilizing dynamic modes—often referred as critical slowing down. This results from the real part of the leading eigenvalue becoming zero as the system closely approaches the bifurcation point (Scheffer et al., 2009).

As illustrated in a pioneering work by S. Carpenter (Carpenter and Brock, 2006) on EWI-based monitoring of shallow lake ecosystems, the onset of the critical slowing down can lead to a drastic increase in the variance of a variable of the system. More recently, the skewness (Guttal and Jayaprakash, 2008), kurtosis (Biggs et al., 2009) and the autocorrelation function (Livina and Lenton, 2007) of a system variable have been shown to capture relevant information on the increased loss of stability close to the bifurcation point.

The literature in early warning detection is rapidly expanding and their potential applications within a broad range of real world problems is being discussed (Scheffer et al., 2009, 2012). Within this contextual frame, it becomes natural to inquire about the possible long-term complex behavior that would arise if EWIs are repeatedly used to control systems potentially undergoing regime shifts. As far as we know, this question remains open, in the light of earlier and further recent advanced methods in the construction of EWI’s (Dakos et al., 2012).

In order to tackle this question, we propose to analyze the dynamics of a system represented by (19) subject to a control process. The basic idea is that such a steering control should: 1) re-establish the stability of the system before it crosses a critical point, or 2) re-establish a lost initial system regime once a critical point has been crossed, to then let the system evolve again towards criticality.

Let us explore this idea by considering a control system which reacts upon the behavior of an EWI, denoted by $I_\tau$, whose construction is based on information about the past states of the system within a sliding observation time window of size $\tau$. In other words, we are interested in analyzing the coupled dynamics of (19) with an evolution equation for $\lambda$, of the form

$$d\lambda = \Phi(I_\tau, \alpha)dt + dW_\lambda$$

Here we require the control functional $\Phi$ to operate a change of sign, at the level of $d\lambda$, once $I_\tau$ crosses a reference value $V_R$. In order to allow for differences in the degree of sensitivity of the control system, let us consider the control response to changes in the ratio $I_\tau/V_R$ as modulated by a parameter $\alpha$. Moreover, let us assume that the control response may exhibit random variability as a result of the influence of many uncorrelated factors. Thus, we denote by $W_\lambda$ a Gaussian Wiener process with standard deviation $\omega_\lambda$.

In the domain of application of EWIs, relevant cases are those where the control variable $\lambda$ is driven by human interventions. In many of these instances, the evolution of the system towards criticality is the result of an “exploitation” process, that is sought to be intensified and maintained, while avoiding a system regime shift. Consider for instance catching rates in sustainable fisheries (Hilborn, 2007), agriculture related leaching rates of phosphorus into a shallow lake, whose oligotrophic regime is to be preserved (Scheffer et al., 1993), or the increase in the autocatalytic production of chemical species in a well-stirred reactor (Nicolis and Nicolis, 2007), where a
low entropy production regime may be desirable. Consequently, we characterize the dynamics of system (19)-(20) in terms of two phases: exploitation if $\Phi > 0$ and recovery if $\Phi < 0$.

Central questions for us are:

\textbf{i}) The role of the time-scale $\tau$, considered for the construction of the EWIs, in shaping the emergence of control properties in a system (19)-(20).

\textbf{ii}) The long-term complex dynamics that can result from the temporal non-locality introduced by the EWI into the dynamics of the coupled system (19)-(20). In other words, in the long-term, the control exerts changes on the system on the basis of the past dynamics of the system and its control as a whole. This opens the possibility to observe different complex behaviors, as the effective dimension of the system-control is augmented by its dependency on the past.

\textbf{iii}) The efficiency of the control process in terms of: its capacity to increase and maintain exploitation in a selected regime as a function of the observation window $\tau$, of the control response sensitivity $\alpha$ and of the variance of stochastic perturbations $\omega_x$ and $\omega_\lambda$.

In Sec. II we address these questions, by exploring numerically the performance of control on minimal dynamical systems of generic character, that reproduce the prototypical phenomena of bistability and oscillations. Each of these controlled systems is first studied in absence of noise, in order to reveal the role of the observation time scale in shaping the leading deterministic dynamics. Upon the basis of the insights thereby generated, for each system, we introduce a suitable measure of control efficiency. Subsequently, we address the influence of the variance of noise, for different observation time windows and different degrees of control response, on the efficiency of both systems under consideration. Finally, in Sec. III we summarize our main results and conclusions.

12 Normal forms and control

Regardless of the complexity of its dynamics, as a system evolves further into the vicinity of a bifurcation point, there is a decrease in the number of degrees of freedom necessary to effectively describe the dynamics. This remarkable fact can be understood as a direct consequence of the gradual run out of the fast-relaxation dynamic modes that occurs along with the critical slowing down. A corner stone in local bifurcation theory is the application of the centre manifold method (Guckenheimer and Holmes, 2013) to identifying canonical descriptions of the nonlinear behavior around a bifurcation point, in terms of a reduced number of variables that contain the whole information about the dynamics of the slow leading modes. Such simple representations—so called normal forms—describe the long-term behavior of the normalized amplitude of the stable solutions that emerge at criticality. The importance of normal forms lies on the fact that the dynamics of different types of systems, around the critical point, can be mapped one-to-one continuously onto the one displayed by a normal form (Nicolis, 1995). This property—a property known as topological equivalence—allows to define classes of universality, where near the critical point, the dynamics of systems belonging to the same class can be mapped onto each other by a change of variables preserving the direction of trajectories. Naturally, a simple and most representative element in a class of universality is thus the corresponding normal form.

With the aim of tackling questions \textbf{i)-iii)} above in a general context, we shall focus our analysis of control dynamics on selected normal forms. As an EWI we consider the variance $M^2_{\tau}[\delta x_j] = \frac{1}{\tau} \int_{t-\tau}^{t} \langle \delta x_j(t') \rangle^2 dt'$ of deviations of a system variable around its mean value, within the time window $\tau$, i.e. $\delta x_j = x_j - \frac{1}{\tau} \int_{t-\tau}^{t} x_j(t') dt'$. Consistently with the minimal character of the
Figure 10: Plot of a fold bifurcation diagram, as provided by the stationary solutions of the cusp normal form 23, plot of the branches formed by the critical values of $\lambda$, and plot $\Phi/k_\lambda$ vs. $M^2_c/V_R$. a) plot of a fold bifurcation diagram, as provided by the stationary solutions of the cusp normal form 23, as a function of parameter $\lambda$ and for $\mu = 3/2$. The solid lines correspond to the stable branches and the dashed line to the unstable one. Unstable and stable branches meet and anhihilate each other at the saddle-node bifurcation points $f_1$ and $f_2$; b) plot of the branches formed by the critical values of $\lambda$ corresponding to both $f_1$ and $f_2$ bifurcation points, as a function of a second parameter $\mu$. Both branches converge to the cusp bifurcation point CB at $\mu = 0$. Finally, plot $\Phi/k_\lambda$ vs $M^2_c/V_R$, for $\alpha = 1$ (dot), $\alpha = 2$ (dot-dot-dash), $\alpha = 5$ (dot-dash), $\alpha = 10$ (dash) and $\alpha = 20$ (solid).
normal form description, let us consider a simple expression for the control functional in (20), of the form

$$\Phi(\tau, \alpha) = k_\lambda \left( 1 - 2 \frac{M_x^2[\delta x_j]^\alpha}{M_x^2[\delta x_j]^{\alpha + V_R}} \right), \quad k_\lambda, \alpha > 0 \quad (21)$$

where $k_\lambda$ corresponds to the maximum exploitation and recovery rates. For low values in $\alpha$ both the exploitation and recovery phases develop weakly as $M_x^2[\delta x_j]$ crosses the threshold value $V_R$. As $\alpha$ is arbitrarily increased, the functional $\Phi$ sharply approaches a Heavyside function (see (Fig. (10)c)) –which emulates a full blown response of the control system (20) to a threshold overshooting

$$\Phi(\tau, \alpha \to \infty) = \begin{cases} 
  k_\lambda \frac{V_R}{M_x^2[\delta x_j]} > 1 \\
  0 \quad \frac{V_R}{M_x^2[\delta x_j]} = 1 \\
  -k_\lambda \quad \text{otherwise}
\end{cases} \quad (22)$$

Here, a comment is in order with regard to the choice of the variance of a system variable as an EWI. As discussed in (Dakos et al., 2011), in the more general case where the standard deviation of the random perturbations is a time dependent function, the variance of a system variable may not constitute a reliable EWI. Consequently, we shall restrain ourselves to the case where both standard deviations $\omega_x$ and $\omega_\lambda$ in (19)-(20) are constants.

12.1 The cusp-control system (CCS)

As pointed out in (Scheffer et al., 2009; Boettiger and Hastings, 2012), the phenomenology observed in so called catastrophic regime shifts in different types of systems can be suitably assimilated with a saddle-node bifurcation. As a parameter evolves, a saddle-node bifurcation occurs when a stable (node) and an unstable (saddle) fixed point solution, to the deterministic evolution law of the system, approach each other and anihilate upon meeting. This entails the destabilization of the system, which is thus led to an abrupt runaway –this constitutes a catastrophic regime shift.

However, since in real systems unbounded runaways are ruled out, saddle-node bifurcations are typically found in cases where the branch formed by the system fixed points folds upon itself, within a range of values in the control parameter $\lambda$ (see Fig. 10a). The presence of such a folding entails the existence of a region where two stable solution branches coalesce along with an unstable one. In this picture, the unstable branch marks the boundary line between the two basins of attraction, associated to both stable branches. In general, the distance between the bifurcation points at the extremes of the s-shape folding ($f_1$ and $f_2$ in Fig. 10b) is determined by a second parameter $\mu$. Examples of such a bistability are reported in different types of systems, ranging from laser and cell division, to ecosystems and climate (Scheffer et al., 2009).

In order to explore the question of bifurcation control in the context of bistability, let us consider the cusp normal form

$$F(x, \lambda) = x(\mu \pm x^2) + \lambda \quad (23)$$

The solutions to the stationary form $F(x_s, \lambda) = 0$ (with the sign minus in (23)), as a function of $\lambda$, correspond to those plotted in Fig. 10a. The position of the saddle-node bifurcation points $f_{1,2}$ are determined by the relation $f_{1,2} = \{(\lambda, \mu)| \lambda = \pm \frac{2}{\sqrt{3}} \mu^{3/2}\}$. Accordingly, both branches approach each other and merge as $\mu$ tends to zero, at the so-called cusp bifurcation point (Fig. 10b).
Figure 11: Plot of a CCS deterministic trajectories in the \((\lambda, x)\)-plane, plot \(x\) vs time, and plot of \(\Phi/k_{\lambda}\) vs \(\lambda - \lambda_c\).

For \(\tau = 10\) (red), \(\tau = 11\) (green) and \(\tau = 35\) (blue), a) plot of a CCS deterministic trajectories \((\omega_x = 0, \omega_{\lambda} = 0)\) in the \((\lambda, x)\)-plane (in black lines the bifurcation diagram of the cusp normal form); b) plot \(x\) vs time (in orange the critical value \(x_c\)) and c) Plot of \(\Phi/k_{\lambda}\) vs \(\lambda - \lambda_c\), as related to the system trajectories in panels a) and b), for \(\tau = 11\) and \(\tau = 35\). Here, values \(\Phi > 0\) and \(\Phi < 0\) are considered as exploitation and recovery, respectively. Parameters and initial condition: \(\mu = 3/2\), \(k_{\lambda} = 1/10\), \(\alpha = 2\), \(V_R = 1/100\). \(x(0) = -3/2\), \(\lambda(0) = -5\). Time step \(dt = 5 \times 10^{-3}\), numerical integration over \(5 \times 10^5\) time steps.
A similar behavior occurs when changing the sign in the second term in (23), which amounts to inverting the s-shaped folding.

We carry out the numerical integration of the variance-based cusp-control system (CCS) (19), (20), (21 and 23) via a standard Euler-Maruyama approximation (Gardiner, 2010).

12.1.1 CCS deterministic dynamics

In order to illustrate the dynamics of the CCS in absence of noise \( (\omega_x = 0, \omega_\lambda = 0) \), let us consider exploitation as initially occurring along the negative branch in Fig. 10a, a moderate degree of control response \( (\alpha = 5) \) (see Fig. 10c) and a small variance reference threshold \( V_R = 1/100 \).

According to the numerical results, we distinguish three basic long-term CCS behaviors:

(B1) **Exploitation runaway**: If the time window is small as compared to a critical value \( \tau_c \), the system crosses the bifurcation point \( f_1 \) and a regime shift occurs. As the system is expelled onto the upper branch, the control mechanism reacts to the overshooting of the reference value \( V_R \). This triggers a recovery stage \( (\Phi < 0) \) along the upper branch. However, as the system evolves along the upper branch, it eventually losses completely the “memory” on the original low branch regime. As a result, the variance decreases gradually below \( V_R \), recovering is interrupted and the CCS switches to runaway exploitation (red lines in Figs.11a,b).

(B2) **Hysteretic pathways**: When the observation window is now greater than the critical value \( \tau_c \), a transition occurs, from runaway exploitation to long-term control. For specific \( \tau \)-ranges, the system crosses the bifurcation point \( f_1 \) and it starts recovering along the upper branch. However, differently from B1, the upper branch recovering continues further and the bifurcation point \( f_2 \) is crossed. Thus, the system undergoes again a regime shift towards the original lower branch. As the system stability increases and information about the upper branch is lost, the CCS eventually switches to exploitation, beyond \( f_1 \) (green lines in Figs.11a,b). This exploitation-recovery process is maintained in the long-term. Processes describing cycles along the zone of bistability are commonly referred as hysteretic. Figure 11c depicts the long-term hysteretic exploitation-recovery process, as a function of the proximity to the critical value \( \lambda_c \) corresponding to the bifurcation point \( f_1 \).

(B3) **Branch-confined pathways**: For large \( \tau \) values, escapes to the upper branch are supressed and the system remains oscillating aperiodically without crossing the bifurcation point \( f_1 \) (blue line, Figs.11a,b). This aperiodic dynamics is predictable in the sense that the average growth of an initially small deviation around a reference state (Nicolis and Prigogine, 1989) is sub-exponential (not shown). The increase in complexity of the dynamics occurring along with \( \tau \) is illustrated by the inset of Fig. 11a and by Fig. 11c (blue lines). The overall range of exploitation and recovery becomes larger as \( \tau \) is increased Fig. 11c. In contrast with case B1, the evolution towards the bifurcation point is marked by short exploitation-recovery-exploitation cycles Fig. 11c, where effective recovery avoids a system regime shift. For certain \( \tau \)-ranges long-term intermittence between behaviors B2 and B3 arises (not shown).

It is worth noticing from Fig. 11a that the regime shifts occur past and not at the crossing of the critical point \( (x_c, \lambda_c) \) –following apparent extensions of the lower stable branch, beyond the critical point. Such an “inertia” effect is a result of the critical slowing down, where the dynamics of \( x \) becomes ’slaved’ by the slower dynamics of \( \lambda \).
Figure 12: Plot of $\epsilon$ vs $\tau$ for deterministic CCS trajectories and for different values of $\omega_x$. For $\alpha = 1$ (yellow), $\alpha = 5$ (green) and $\alpha = 20$ (magenta) and $\omega_\lambda = 0$, a) plot of $\epsilon$ vs $\tau$ for deterministic CCS trajectories ($\omega_x = 0$). Similar plots for stochastic trajectories with b) $\omega_x = 5$, c) $\omega_x = 10$ and d) $\omega_x = 25$. Parameter values, initial condition and numerical integration as in Fig. 11. Here, each $\epsilon$ value has been computed for $1 \times 10^6$ time steps.

12.1.2 CCS control efficiency

A meaningful way to characterize further the CCS dynamics is to quantify the control capacity to increase and maintain long-term exploitation within a selected stable branch, as a function of the model parameters. Accordingly, we quantify the efficiency as

$$\epsilon(\tau, \alpha) = \lim_{T \to \infty} \frac{\omega_R^2}{T M R^2} \int_0^T \frac{\Phi(\tau, \alpha) H(\Phi(\tau, \alpha)) H(-x(t))}{k_\lambda} dt$$

(24)

$H$ denotes a Heaviside function, whose value is zero for negative argument or one otherwise. The product of the Heaviside functions in the integral plays the role of an AND boolean operator, entailing that the contributions to the time average of $\Phi$ are considered only if $\Phi$ is positive and $x$ negative. The variance of deviations from the initial to the final time $T$, in the denominator of (24), aims at amplifying differences between hysteretic (B2) and branch-confined (B3) control regimes. Without loss of generality, hereafter we set the reference value $\omega_R^2 = 1$ in (24).

In the next subsections we focus on the behavior of the efficiency as a function of the size of the observation window $\tau$, of the degree of control response $\alpha$ and of the variance of random fluctuations $\omega_x$ and $\omega_\lambda$.

12.1.3 Deterministic CCS efficiency

Figure 12a summarizes the behavior of the efficiency (24) over the deterministic CCS trajectories corresponding to different values in $\tau$ and $\alpha$. Efficient control rises sharply as the observation window is increased above an $\alpha$-specific threshold $\tau_c$—since below this value the CCS undergoes exploitation runaway (B1). Above $\tau_c$, the behavior of the efficiency becomes highly dependent on the degree of control response. For a low control response ($\alpha = 1$), the efficiency exhibits
a quasi-constant plateau for $\tau > \tau_c$. This plateau consists of hysteretic cases (B2). In cases of moderate ($\alpha = 5$) and high ($\alpha = 20$) degree of control response, the behavior of the CCS efficiency becomes non-trivial. For values $\tau \gg \tau_c$ the efficiency is remarkably enhanced (see the appearance of a second plateau occurring for $\tau \geq 25$ in Fig. 12a), as a result of the suppression of the CCS hysteretic control pathways, in favour of branch-confined recovery-exploitation cycles (B3). For intermediate values $\tau > \tau_c$, a combination of pathways (B2) and (B3) is observed —where low efficiency values correspond to hysteretic control pathways (B2). Such a $\tau$-dependent selection of control pathways is further illustrated by the histogram of the $x$ variable in Fig. 13a, for the high control response case. It shows that the dynamics of the CCS becomes confined to $x < 0$ values as $\tau$ is slightly shifted, which explains the alternance of low and high efficiency ranges appearing at intermediate $\tau$ values. Comparing the different control response cases in Fig. 12a, it can be observed that the moderate one is the most efficient for $\tau \gg \tau_c$.

### 12.1.4 Stochastic CCS efficiency

Let us address now the case where the evolution of $x$ is marked by continuous random perturbations (i.e., $\omega_x \neq 0, \omega_\lambda = 0$ in Eqs. (19) and (20)). Panels b) to d) in Fig. (12) illustrate the influence of noise on the CCS efficiency. In the case of low control response, the behavior of the efficiency with $\tau$ appears to be independent of the standard deviation of noise $\omega_x$. In contrast, for moderate and high control responses, the overall efficiency becomes enhanced by an increase in the intensity of noise, in the sense that low efficiency values tend to disappear while high efficiency occurs all the way above the $\tau_c$-value (compare Fig. 12a with Figs. 12b–d). However, if the variance of noise is marginally large (Fig. 12a-c), the high control response case is less efficient as compared with...
Figure 14: Plot of $\varepsilon$ vs $\tau$ for deterministic CCS trajectories and for different values of $\omega_\lambda$.
For $\alpha = 1$ (yellow), $\alpha = 5$ (green) $\alpha = 20$ (magenta) and $\omega_x = 0$, plot of $\varepsilon$ vs $\tau$ for stochastic CCS trajectories with a) $\omega_\lambda = 5$, b) $\omega_\lambda = 10$ and c) $\omega_\lambda = 20$. In d), similar plots for $\omega_\lambda = 25$ and $\omega_x = 20$. Parameter values, initial condition and numerical integration as in Fig. 11. Here, each $\varepsilon$ value has been computed for $1 \times 10^6$ time steps.

the moderate one. For strong noise (Fig. 12d), both moderate and high control response cases converge to a similar efficiency value for large $\tau$.

Since, for high and moderate response cases, low efficiency values tend to disappear when increasing intensity noise, it is natural to inquire about the influence of strong noise on hysteretic pathways. This effect is shown by the histogram of $x$ Fig. 13b for a single realization with $\alpha = 20$. Notice the similarity between panels (a) and (b), which illustrates the equivalence in the control pathway selection that occurs either by changes in $\tau$ (Fig. 13a) or by an increase in the standard deviation of the random fluctuations $\omega_x$ (Fig. 13b).

Interestingly enough, we observe a similar effect of efficiency enhancement and uniformization in the case of noise at the level of the control system (i.e., $\omega_x = 0$, $\omega_\lambda \neq 0$) Fig. 14a-c, as compared with the deterministic CCS (Fig. 12a). However, in this case the emergence of efficient control occurs for larger values in $\tau$, as the intensity of noise increases. Finally, in the case of strong noise at the level of both the state variable $x$ and control (i.e., $\omega_x \gg 0$, $\omega_\lambda \gg 0$) Fig. 14d, the case of high control response becomes the most efficient one, in comparison with the ($\omega_x \neq 0$, $\omega_\lambda = 0$) cases (Fig. 12d).

12.2 The Hopf-control system (HCS)

A relevant and also frequently observed type of regime shift consists in the onset of time periodic behavior, as a stationary system regime becomes unstable at a threshold value in a system parameter. Examples of processes and systems where such phenomenon is present are for instance, chemical clocks at the mesoscopic (McEwen et al., 2010) and macroscopic (Epstein and Pojman, 1998) scales, cellular biochemical cycles (Goldbeter, 1997), prey-predator population dynamics (Murray, 2011), shallow lake ecosystems (Scheffer, 2009), the ocean-atmosphere system (Vallis, 1988), or non-equilibrium economics models (Hallegatte et al., 2008). All models accounting for
the emergence of oscillatory behavior involve at least two variables and in all cases it has been possible to assimilate the mechanisms underlying the critical onset of oscillations to a Hopf bifurcation (Guckenheimer and Holmes, 2013). In what follows we focus on the dynamics of the coupled system of control (20)-(21) and the deterministic evolution law in (19), given by the Hopf normal form –which we refer hereafter as the Hopf-control system (HCS). For illustrative purposes and without loss of generality, we consider the supercritical version of the Hopf normal form

\[ F_1(x_1, x_2, \lambda) = \lambda x_1 - x_2 - x_1(x_1^2 + x_2^2), \quad F_2(x_1, x_2, \lambda) = \lambda x_2 - x_1 - x_2(x_1^2 + x_2^2) \]  

(25)

The stationary solution \( x_1 = 0, x_2 = 0 \) to the Hopf normal form (25) losses stability as \( \lambda \) approaches zero along the negative axis. Above the critical value \( \lambda_c = 0 \), the system variables \( (x_1, x_2) \) describe stable oscillations whose amplitudes increase as \( \sqrt{\lambda} \).

Here, the control functional (21) is fed by either of both system variables \( (x_1, x_2) \), whose dynamics is subjected to stochastic perturbations with same standard deviation \( \omega_x \).

12.2.1 Deterministic HCS dynamics

For an initial condition around the stationary solution \( x_1 = 0, x_2 = 0 \) and for an intial value \( \lambda < 0 \), Figs.15a-c show the deterministic dynamics of the \( x_1 \) component under the influence of control for different values of the time window \( \tau \). In complete absence of control, the linear growth in the value of \( \lambda \), beyond the critical point, leads to oscillations of increasing amplitude \( \sqrt{\lambda} \). In contrast, for small values \( \tau \gtrsim 0 \) the amplitude of oscillations becomes constant (Fig. 15a). In this situation, both recovery (\( \Phi < 0 \)) and exploitation (\( \Phi > 0 \)) phases succeed each other rapidly, while the control variable always remains above the critical value \( \lambda_c = 0 \) (Fig. 15a’). A further increase in \( \tau \) induces a temporal alternance of small and large oscillation amplitudes (Fig. 15b), as the recovery and exploitation phases lead the system back and forth accross \( \lambda_c \) (Fig. 15b’). If \( \tau \) is large enough, oscillations are temporally suppressed (Fig. 15c), during a time interval that increases with \( \tau \). This
situation results from the fact that the \( \lambda \)-interval covered by the recovery and exploitation phases becomes enlarged (Fig. 15c’). In contrast with the CCS Fig. 23c, no substantial increase in the complexity of the dynamics is observed as the time window \( \tau \) is arbitrarily increased.

### 12.2.2 Deterministic and stochastic HCS efficiency

In a similar vein as for the CCS, we define the efficiency of the HCS as the capacity to increase and maintain long-term subcritical exploitation, while keeping the oscillation amplitudes to a minimum. Thus we write the efficiency as

\[
\epsilon(\tau, \alpha) = \lim_{T \to \infty} \frac{\omega_\lambda^2}{TM^2} \int_0^T \frac{\Phi(\tau, \alpha) H(\Phi(\tau, \alpha)) H(-\lambda(t))}{k_\lambda} dt
\]

where \( H \) denotes a Heavyside function, such as defined for (24).

As shown by Figs.15a’–c’, subcritical exploitation increases with the size of the observation window \( \tau \). However, it is also observed that the amplitude of oscillations along the recovery phase also increases with \( \tau \) (Figs.15a–c). Therefore, an optimum is expected to occur in the efficiency (26) as a function of \( \tau \). This situation is actually illustrated in Fig. 16a for different degrees of control response. Notice that compared to the efficiency of the deterministic CCS Fig. 24a, the sharp rise towards efficient control occurs at a small value \( \tau_c \), above which efficiency gradually decays for larger \( \tau \) values.

In presence of noise at the level of the system variables \( (x_1, x_2) \) (i.e. \( \omega_x \neq 0, \omega_\lambda = 0 \)), it is observed that the efficiency increases along with \( \tau \), for the moderate and high control response cases, while for low control response it remains at low practically constant value (Fig. 16b). Similarly as for the CCS (Fig. 12), the efficiency is in overall enhanced by the intensity of noise Fig. 16b–d. Moreover, the behavior of the efficiency is practically the same for moderate and high
control response cases, with the exception of the strong noise case Fig. 16d, where high control response is highest for arbitrary large \( \tau \) values.

When considering the effect of stochastic perturbations on the control dynamics (i.e. \( \omega_x = 0, \omega_\lambda \neq 0 \)) Figs.17a–c, the behavior of the efficiency remains qualitatively the same as in the purely deterministic case Fig. 16a. Regardless the intensity of control fluctuations, a maximum in efficiency is observed at an approximately same \( \tau \) value. The efficiency associated with high and moderate control responses exhibit a similar dependency on \( \tau \). In both cases, the maximum in efficiency is higher as compared to the low response one. However, the highest efficiency peaks are slightly lowered when augmenting the intensity of noise at the level of control. For all the control response cases, when increasing \( \tau \) the corresponding efficiencies tend to converge to a same low value, independently of the intensity of noise in control.

Finally, in presence of strong fluctuations at the level of the dynamics of both the system and the control (i.e. \( \omega_x \gg 0, \omega_\lambda \gg 0 \)), the efficiency rises towards a plateau for larger values of \( \tau \). In this case it is observed that the presence of strong fluctuations enhances the control efficiency and to a greater extent in the cases of high and moderate control response.

13 Summary and conclusions

In this work we addressed the variance-based control of systems evolving towards criticality. As testbed models we considered the cusp and Hopf normal forms, displaying the prototypical phenomena of bistability and oscillations, respectively.

In relation to the core questions (i)–(iii) exposed in Sec. I, our main results can be summarized as follows:

i) In absence of noise, effective control in both the cusp and the Hopf systems rises above a threshold value in the size of the observation time window.
ii) In the cusp-control system, changes in the observation time window provide a means to selecting among different control pathways, namely, hysteretic, branch-confined control, as well as an intermittent combination of both. The branch-confined regime becomes dominant when considering a large observation window. This entails an increase in the complexity of the dynamics, in the form of aperiodic control-system trajectories. In the case of the Hopf-control system, the crossing of the bifurcation point could not be avoided, for the range of parameters explored. Instead, as the observation time window is enlarged, exploitation and recovery phases occur for longer periods. This gives rise to the alternance of time intervals where oscillations are suppressed, followed by amplified oscillations.

iii) In both systems, random fluctuations, at the level of the system variables, enhance the efficiency of control, and especially for large observation time windows. In the case of the cusp-control system, the intensity of noise induces a control pathway selection, similarly as in the deterministic case when varying the observation time window. From the deterministic to the strong fluctuations cases, the efficiency of the cusp-control system tends to increase for large values in the size of the observation window. In the Hopf-control system, an increase in the intensity of noise induces a reduction in the amplitude of oscillations, which entails an increase in the control efficiency. Random fluctuations at the level of the control dynamics exert opposite influences on the control efficiency of the cusp and Hopf control systems. In the former it increases, while for the latter it tends to diminish the peak of efficiency. For both cusp and Hopf control systems, the combination of fluctuations in the control and in the system dynamics amplifies the efficiency in particular for large observation time windows. Regarding the interplay between the efficiency, the degree of control response, the intensity of noise, and the observation time window, we observe: in both systems, the low control response case exhibits a low efficiency, independently of the intensity of noise and observation window. When increasing the variance of fluctuations in the dynamics of the cusp system, a moderate control response leads to a higher efficiency; the opposite situation occurs when considering fluctuations on the control dynamics. In the Hopf system, the moderate and high control response perform very similar, regardless the intensity of noise and size of the time window. In this situation, for both moderate and high response cases, the efficiency converges to the lower value associated with a low control response, as the observation time window is increased.

For the systems here addressed, it remains to be studied the influence of other parameters, such as the rate of change in the control system ($k_\lambda$), the control reference parameter ($V_R$), as well as alternative approaches towards bifurcation, such as variation of the parameter $\mu$ in the cusp map—which would induce flickering patterns at the vicinity of the critical point.

Potential applications of this approach to real world systems include ecosystem models, nonequilibrium mesoscopic physico-chemical systems or non-equilibrium models in economics, among others. Similarly, it is of interest to extend this work to address the control properties associated to different alternative early warning indicators. Moreover, our analysis could be extended to address the control of coupled systems, forming networks whose nodes consist in dynamical systems subject to control.

Certainly, the structure of the evolution equations of real world models, involving a large number of parameters, lacks the degree of symmetry that confers to normal forms their characteristic simplicity. Moreover, not every dynamical system exhibiting a bifurcation belongs to a class of universality. It is thus important to address further the relation between the observation time scales leading to effective control and the geometrical properties, such as the curvature, of the system solution branches, while considering different early warning indicators.
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Summary & Discussion

Three different classes of transition functions have been investigated. Most importantly, one needs to distinguish favourable and unfavourable ones. Obviously, in terms of adaptation and mitigation, transition are favourable.

Therefore, in Part I of this deliverable, the theoretical basis for transition functions towards an climate change adaptation state are established (also in terms of adaptation costs). The economic value of urban landscape derives from the (potential) income that can be generated after providing for expected damage costs. E.g. damage from coastal flooding diminishes income and may necessitate one of several adaptation options: protect, accommodate, or retreat. If urban structures cannot be maintained without the continual input of external funds, the abandonment of urban land use is a last resort. Uncertainty is reflected by the probability of transition as a function of economic value. The shape of the curve, in turn, is subject to the interplay of risk aversion and negligence.

Urbanisation is a predominant development in the 21st century. Since cities represent singularities in space, i.e. comprising extreme densities of people and assets, it is highly relevant to study cities and urbanisation in the climate change complex. Most importantly, adaptation in the form of retreat affects current cities and future development. Thus, in Part II patterns of urban growth have been analysed in order to characterise future urbanisation. Therefore, land cover transition functions in the form of logistic regressions (sigmoid function) based on parameters such as existing urban areas, distance to shore, elevation, slope, etc. have been employed. The identified local urbanisation scenarios scenarios allow estimating the additional urban area under risk. The provided parameters represent a database of transition functions for the considered case studies.

Nevertheless, one can also think of unfavourable transitions. Exemplarily, a system exhibiting a bifurcation point is considered in Part III. As the state approaches the unwanted bifurcation point, the variance increases which in turn is used to control the system so that it does not trespass the bifurcation point, but stays close. This closeness could be understood as a variant of self-organised criticality and may be relevant in economic terms, such as optimal efficiency. The results provide insights on the interplay between the time-scale for the system observation, the degree of sensitivity of the control feedback, and the intensity of the random perturbations in shaping the long-term control efficiency.

In summary, a library of standardised adaptation and transition functions has been assembled. Like a library of books, it is never complete but only represents a collection of most important pieces. The database of generalised impact and transition functions will be complemented by transition factors in RAMSES WP8. The work done within Task 1.4 is or will be submitted to scientific journals for publication.

Transition is a very broad concept and this make it difficult to define transition factors and to be precise with the transition functions. Perspectively, in RAMSES WP8 transition factors are being identified, e.g. it can be an option to link the hazard events with the “transition status or stage”. The transition function can be applied in an adaptation assessment. The transition classification can be derived from established adaptive capacity-profiles, i.e. defined by the status of the adaptive capacity.

The probability of such a transition depends on factors like flood, storm or heat wave frequency or duration. Therefore, in RAMSES WP8 the transition function will be defined by the link between the number of events occurred and the degree of adaptive capacity. Collecting data from the city case studies related to the historical number of events and evaluating the adaptive capacity of the city in different years, enables to estimate the degree of the transition of the city.

Alternatively, the land use change can be linked with the degree of transition. In the stake-
holder dialogue the stakeholder identify the land use as a transition factor (WP8), so it is justified to take the land use change as a transition factor.
References


